Mathematics: analysis and approaches
Standard level
Paper 2

Tuesday 1 November 2022 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

12EP01

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

   The following table shows the Mathematics test scores (x) and the Science test scores (y) for a group of eight students.

Mathematics scores (x)	64	68	72	75	80	82	85	86
Science scores (y)	67	72	77	76	84	83	89	91

   The regression line of y on x for this data can be written in the form $y = ax + b$.

   (a) Find the value of a and the value of b. [2]

   (b) Write down the value of the Pearson's product-moment correlation coefficient, r. [1]

   (c) Use the equation of your regression line to predict the Science test score for a student who has a score of 78 on the Mathematics test. Express your answer to the nearest integer. [2]

   (a) $y = 1.01x + 2.45$ $a = 1.01$ $b = 2.45$
       A1   **A1**

   (b) r is 0.98 **A1**

   (c) $y = 1.01 \cdot 78 + 2.45 = 81.4$ rounding this to:
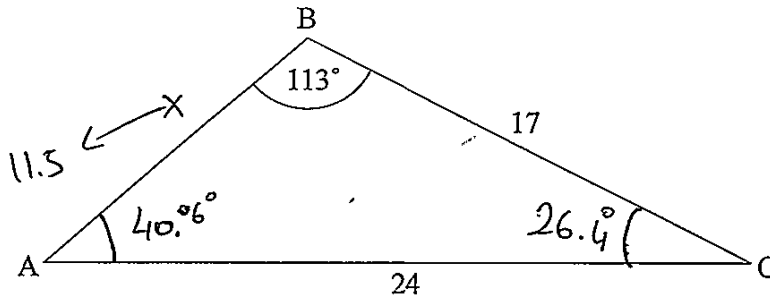 $\boxed{81}$ ✔ 2

$$x^2 = 17^2 + 24^2 - 2 \cdot 17 \cdot 24 \cdot \cos(26.4)$$

2. [Maximum mark: 6]

The following diagram shows triangle ABC, with $AC = 24$, $BC = 17$, and $A\hat{B}C = 113°$.

diagram not to scale

(a) Find $B\hat{A}C$. [3]

(b) Find AB. [3]

(a) $\dfrac{17}{\sin(B\hat{A}C)} = \dfrac{24}{\sin(113)}$

$17\sin(113) = 24\sin(B\hat{A}C)$ **M1**

$\sin(B\hat{A}C) = \dfrac{17\sin(113)}{24}$

$\sin(B\hat{A}C) = 0.652$ **A1**

$\sin^{-1}(0.652) = 40.6°$

$B\hat{A}C = 40.6°$ // **A0**

(b)

$AB^2 = 17^2 + 24^2 - 2 \cdot 17 \cdot 24 \cdot \cos(26.4)$

$AB^2 = 134.0$ **A1**

$AB = \sqrt{134.0} = 11.5$ **M1**
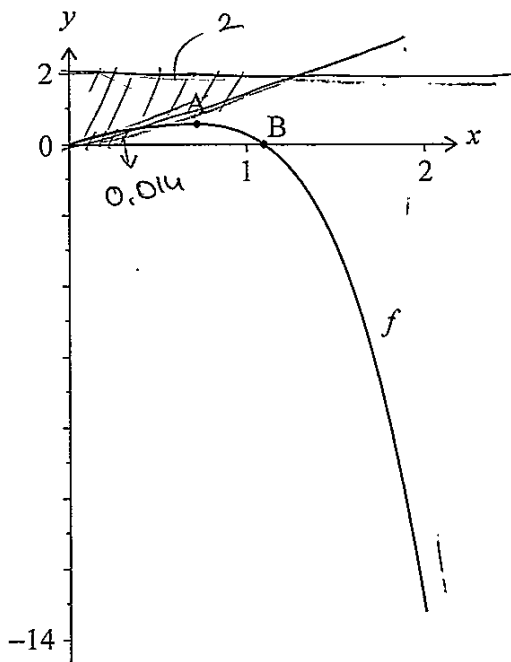
$AB = 11.5$ // **A0**

Turn over

3. [Maximum mark: 6]

The function f is defined as $f(x) = \ln(xe^x + 1) - x^4$, for $0 \le x \le 2$. The graph of f is shown in the following diagram.

The graph of f has a local maximum at point A. The graph intersects the x-axis at the origin and at point B.

(a) Find the coordinates of A. [2]

(b) Find the x-coordinate of B. [1]

(c) Find the total area enclosed by the graph of f, the x-axis and the line $x = 2$. [3]

(a) A̶ ̶n̶a̶ A coordinates = (0.709, 0.64) ·A1· ·A1·

(b) X-coordinate of B = 1.1 A1

(C). 0.014 is the area enclosed by the graph
f and the x-axis AD

4. [Maximum mark: 5]

A geometric sequence has a first term of 50 and a fourth term of 86.4.

The sum of the first n terms of the sequence is S_n.

Find the smallest value of n such that $S_n > 33\,500$.

$S_n = \frac{n}{2}(2 \cdot 50 + (n-1)12.1)$

in GDC

nSolve $(\frac{n}{2}(2 \cdot 50 + (n-1) \cdot 12.1) = 33\,500, n)$

$n = 70.8 \rightarrow$ ~~71~~ // 71 //

The smallest value of n such that $S_n > 33\,500$ is ~~71~~ // 71 //

$U_4 = 50 + 3d$

$50 + 8d = 86.4$

$3d = 36.4$

$d = 12.1$ //

$d - $

A0

Turn over

5. [Maximum mark: 7]

The population of a town t years after 1 January 2014 can be modelled by the function

$$P(t) = 15\,000e^{kt}, \text{ where } k < 0 \text{ and } t \geq 0.$$

It is known that between 1 January 2014 and 1 January 2022 the population decreased by 11%.

Use this model to estimate the population of this town on 1 January 2041.

initial $= 15.000$ — **A1**

$15.000 \times \dfrac{11}{100}$

$P(t) = 15000e^{kt \times 27} =$

in 8 years time, 1650 ~~people~~ people **A1**

are decreased — **A0**

$P(8) = 15000 \cdot e^{k \times 8} = 1650$ — **M1**

$k = 0.27$ — **A0** $P(27) = 15000 \cdot e^{-0.27 \times 27} =$ — **M1**

$10.2349 \rightarrow$ Approximately 10 people left in this town by 1 January 2041

A0

6. [Maximum mark: 6]

Consider the expansion of $\dfrac{(ax+1)^9}{21x^2}$, where $a \neq 0$. The coefficient of the term in x^4 is $\dfrac{8}{7}a^5$.

Find the value of a.

to find the coefficient of x^4

$\dfrac{(ax+1)^9}{21x^2}$ $\binom{7}{3}\dfrac{7!}{4!\,3!} = \dfrac{7 \cdot 6 \cdot 5 \cdot 4!}{4!\,3!} = 35$

x^7

$\dfrac{8}{7}\,a^5 x = 35\,x^4$

$\dfrac{8}{7}\,a^5 = 35$

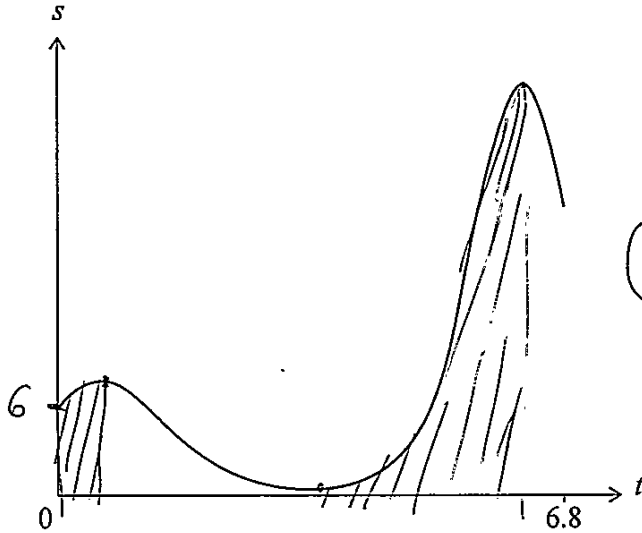$a^5 = \dfrac{35 \cdot 7}{8} = 30.6$

$a = \sqrt[5]{30.6} = 1.98$

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

A particle moves in a straight line. Its displacement, s metres, from a fixed point P at time t seconds is given by $s(t) = 3(t+2)^{\cos t}$, for $0 \le t \le 6.8$, as shown in the following graph.

$$\left(0 \le x \le 0.438\right) \cup$$
$$\left(3.26 \le x \le 6.34\right)$$

(a) Find the particle's initial displacement from the point P.   [2]

(b) Find the particle's velocity when $t = 2$.   [2]

(c) Determine the intervals of time when the particle is moving away from the point P.   [5]

The acceleration of the particle is zero when $t = b$ and $t = c$, where $b < c$.

(d) Find the value of b and the value of c.   [4]

(e) Find the total distance travelled by the particle for $b \le t \le c$.   [3]

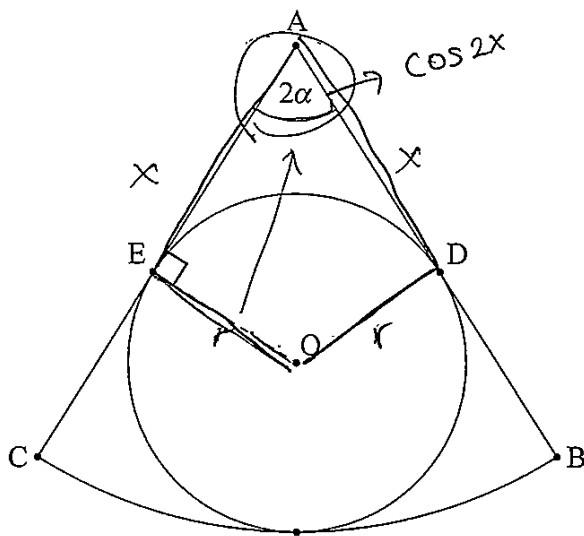**Turn over**

Do **not** write solutions on this page.

8.   [Maximum mark: 13]

The following diagram shows a sector ABC of a circle with centre A. The angle $B\hat{A}C = 2\alpha$, where $0 < \alpha < \dfrac{\pi}{2}$, and $O\hat{E}A = \dfrac{\pi}{2}$.

A circle with centre O and radius r is inscribed in sector ABC.

AB and AC are both tangent to the circle at points D and E respectively.

diagram not to scale

(a)   Show that the area of the quadrilateral $ADOE$ is $\dfrac{r^2}{\tan\alpha}$.   [4]

(This question continues on the following page)

$\tan = \dfrac{\sin \alpha}{\cos \alpha}$

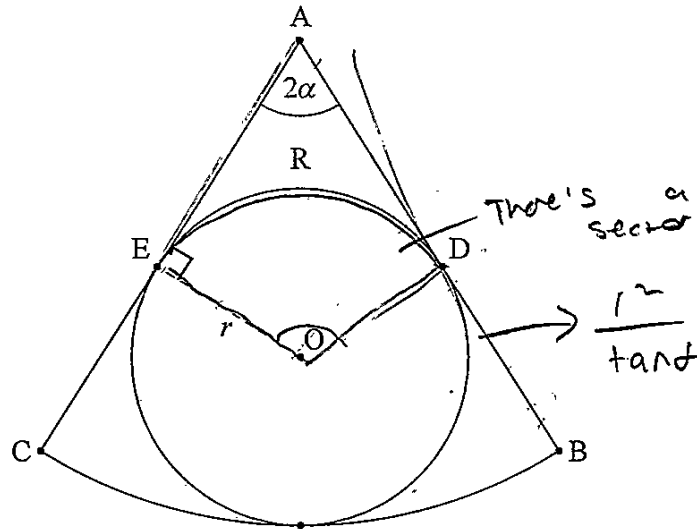$r^2 \cdot \dfrac{\cos \alpha}{\sin \alpha}$

Do **not** write solutions on this page.

(Question 8 continued)

R represents the shaded region shown in the following diagram.

diagram not to scale

(b)　(i)　Find $D\hat{O}E$ in terms of α.

　　　(ii)　Hence or otherwise, find an expression for the area of R. 　　　　　　　　　[5]

(c)　Find the value of α for which the area of R is equal to the area of the circle of centre O and radius r. 　　　　　　　　　[4]

Turn over

Do **not** write solutions on this page.

9. [Maximum mark: 16]

The time worked, T, in hours per week by employees of a large company is normally distributed with a mean of 42 and standard deviation 10.7.

(a) Find the probability that an employee selected at random works more than 40 hours per week. ⤷ 0.574   [2]

(b) A group of four employees is selected at random. Each employee is asked in turn whether they work more than 40 hours per week. Find the probability that the fourth employee is the only one in the group who works more than 40 hours per week.   [3]

(c) A large group of employees work more than 40 hours per week.

(i) An employee is selected at random from this large group.

Find the probability that this employee works less than 55 hours per week.

(ii) Ten employees are selected at random from this large group.

Find the probability that exactly five of them work less than 55 hours per week.   [7]

It is known that $P(a \leq T \leq b) = 0.904$ and that $P(T > b) = 2P(T < a)$, where a and b are numbers of hours worked per week. An employee who works fewer than a hours per week is considered to be a part-time employee.

(d) Find the maximum time, in hours per week, that an employee can work and still be considered part-time.   [4]

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7

(a) initial displacement = when $t=0$
↳ 6 //

(b) ~~crossed out~~

$S'(t) = $ Velocity //

$\frac{d}{dx}\left(3 \cdot (x+2)^{\cos(x)}\right)\Big|_{x=2} = -2.29$ //

(C) $(0 \leq x \leq 0.433) \cup (3.26 < x < 6.34)$

(d) Acceleration → $S''(t)$

$S'(t) = 3(t+2)^{\cos t}$
$U = 3 \quad V = (t+2)^{\cos t}$

$U' = 0 \quad V' = \cos - \sin t \cdot (t+2)^{-\sin t \cos t}$

~~3 sin t~~ $3 \cdot -\sin t (t+2)^{\cos t} + (t+2)^{\cos t} \cdot 0$

$= 3 \sin t (t+2)^{\cos t} = S'(t)$ //

$S''(t) = 0$ will give the b and c values //

total distance travelled:

$$\int_b^c S''(t)\ dt \quad \boxed{\text{A0}}$$

8 (a) 2α

$\cos 2\alpha = \quad \boxed{\text{A0}}$

9

(a) norm cdf (40, 9999, 42, 10.7) = 0.574   ✓ 2

(b) binom cdf (4, 0.574, 0, 0) = 0.032

0.032 × 0.574 = 0.018 //   A0

(c) ~~be~~ (i) norm cdf (40, 55, 42, 10.7) = 0.461   A0

(ii) Binom Pdf (10, 0.461, 5) = 0.23

M1   A1FT   A0